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Correlations in random Ising chains at zero temperature

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Abstract. We present a general method to calculate the connected correlation function of random Ising chains at zero temperature. This quantity is shown to relate to the survival probability of some one-dimensional, adsorbing random walker on a finite interval, the size of which is controlled by the strength of the randomness. The correlation length is exactly calculated for different random field and random bond distributions.

1. Introduction

A perfect crystal is a physical abstraction; in real materials one should always expect impurities and different types of lattice defects. There are important problems in the field of random systems, such as localization in a disordered medium, spin-glass behaviour, diluted magnets etc. In order to obtain a theoretical understanding of these phenomena, magnetic models with quenched disorder have been introduced and studied by different methods (for recent reviews, see [1–3]). Random magnetic models have unusual low-temperature properties. These are a consequence of the complex structure of low-energy metastable states generated by quenched disorder and frustration, which are believed to be the main ingredients of spin-glass behaviour [4]. The simplest system in this field is the random Ising model, which can be experimentally realized as diluted antiferromagnet [5]. Already the one-dimensional model shows interesting features, although its physical quantities are only singular at zero temperature.

Despite its low dimension, the one-dimensional random Ising model is exactly solvable only for several specific types of randomness. One of these is the random-bond Ising chain in a uniform field (RBIM) defined by the Hamiltonian

$$\mathcal{H}_{\mathrm{RB}} = -\sum J_{i,i+1}\sigma_i\sigma_{i+1} - h\sum \sigma_i. \tag{1}$$

Here $\sigma_i = \pm 1$, the exchange integral $J_{i,i+1}$ is equal to $J > 0$ with probability p and to $-J$ with probability $q = 1 - p$, and the bond disorder is quenched. For this model the ground state energy, the zero point entropy and the magnetization have been calculated by Derrida and co-workers [6] (see also in [7]).

Another type of random system is the ferromagnetic Ising model in a random field (RFIM) with the Hamiltonian

$$\mathcal{H}_{\mathrm{RF}} = -J\sum \sigma_i\sigma_{i+1} - \sum h_i\sigma_i. \tag{2}$$

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At $T = 0$ the thermodynamic properties of this model have been calculated for the binary distribution of the random fields [6],

$$P(h) = p\delta(h - H) + q\delta(h + H) \tag{3}$$

as well as for the asymmetric binary distribution [8]:

$$P(h) = \tfrac{1}{2}\delta(h - H_1 - H_0) + \tfrac{1}{2}\delta(h + H_1 - H_0). \tag{4}$$

The first exact results at $T \neq 0$ have been obtained by Grinstein and Mukamel [9] with the diluted symmetric binary distribution,

$$P(h) = \tfrac{1}{2}p[\delta(h - H) + \delta(h + H)] + q\delta(h) \tag{5}$$

in the limit $H \to \infty$. Very recently the nonlinear and higher-order susceptibilities of the model have also been determined [10].

A series of RFIMs with continuous random field distribution has been studied by Luck and Nieuwenhuizen [11–13] at arbitrary temperatures. One of these models is characterized by the diluted symmetric exponential distribution $R(x)\,dx$ as

$$R(x) = \frac{p}{2}e^{-x} + q\delta(x) \tag{6}$$

with $h_i = \tilde{H}x_i$, $\tilde{H} > 0$ and $-\infty < x_i < \infty$.

As far as the correlation functions of random Ising models are concerned, few results are available. Derrida [14] has pointed out that the spin–spin correlation function is not a self-averaging quantity, therefore its average differs from the most probable value [15]. Exact results in the entire temperature range are available for the Grinstein–Mukamel model [9], as well as for the diluted symmetric exponential distribution in equation (6) [12]. Most recently Farhi and Gutmann [16] calculated the zero-temperature correlation function of RFIM with the binary distribution in equation (3). We should also mention a related exact study of the pair-correlation function of a one-dimensional lattice gas model in a random potential at zero temperature [17].

In this paper we study the connected correlation function of random Ising chains defined as

$$\chi(l) = \left[\langle\sigma_i\sigma_{i+l}\rangle - \langle\sigma_i\rangle\langle\sigma_{i+l}\rangle\right]_{\mathrm{av}} \tag{7}$$

where $\langle\ldots\rangle$ denotes the thermodynamic average and $[\ldots]_{\mathrm{av}}$ stands for the quenched average over random variables. The thermodynamic average in equation (7) could be non-zero even at $T = 0$, provided the ground state of the system is highly degenerate. This can be seen in two-dimensional frustrated models without randomness, in which zero-temperature correlations decay either as a power law [18–21] or exponentially [22].

For random Ising chains the correlation function in equation (7) is usually calculated in the transfer matrix formalism [9, 12]. At $T = 0$, however, one may use another approach based on an analysis of the degenerate ground-state configurations. For a given quenched disorder, owing to frustration, there are spins in the system which are 'loose', i.e. they are free to point in any direction. These loose spins may form a domain, the connected correlation function being non-zero, but only if both spins considered belong to the same domain of loose spins. At this point a calculation of the correlation function at $T = 0$ is essentially reduced to an investigation of the size distribution of domains of loose spins, which in turn is equivalent to a one-dimensional random walker problem on a finite interval.

The setup of the paper is as follows. In section 2 we present our method to calculate the connected correlation function for random Ising chains at $T = 0$. In sections 3 and 4 correlation lengths are calculated for RFIM and RBIM, respectively. Finally, the results are discussed in section 5.

2. Connected correlations and their relation to random walkers

In this section we develop a formalism to calculate the connected correlation function for random field Ising models at $T = 0$. It will be shown in Section 4 how these results can be applied for the random-bond problem.

The random fields we consider have a discrete distribution. Furthermore, the possible values of h_i are integer multiples of a unit, denoted by H, i.e. $h_i = m \times H$. For the binary distribution in equation (3), $m = \pm 1$, while for the diluted symmetric distribution in equation (5) we have $m = 0, \pm 1$. Continuous distributions, like that in equation (6), can also be discretized (an example is shown in section 3.3). In this way one can also treat the asymmetric binary distribution in equation (4), provided the ratio of the parameters H_1 and H_0 is rational.

The structure of groundstate configurations of a RFIM with discrete randomness is thoroughly analysed in the literature (see e.g. in [2, 3]). In the weak-coupling limit, when $2J < \min\{|h_i|\} = h_{\min}$, the spins are frozen in the direction of the local fields. For stronger couplings ($2J > h_{\min}$), neighbouring spins tend to align parallel to each other, so that domains of parallel spins are formed. With stronger coupling, the average size of a domain increases but the ground state never consists of one single domain. It can be explained by pointing out that the energy $2J$ necessary to create a domain wall can be accumulated from fluctuations of the random field, even if the local field is arbitrarily small.

The scale of random field fluctuations is characterized by the integrated random field function defined as $H(k) = \sum_{i=1}^{k} h_i$. As an illustration, we draw this function on figure 1 for a given random field distribution, together with the corresponding ground-state configuration of the system at some value of the coupling J. As we see on this figure, the first domain wall is located between spins 3 and 4, at a local maximum of $H(k)$. Indeed, the sum of random field energies $H(6) - H(3) < -2J$ covers the cost of creating a domain wall.

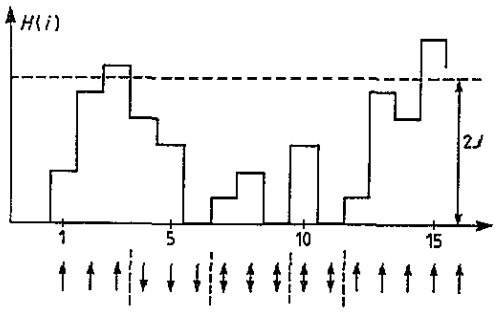

Figure 1. The integrated random field function $H(i)$ for some fixed values of the random field. The corresponding ground state of the system at a given value of the coupling J is indicated below. Here vertical dashed lines denote the possible domain wall positions. State of spins in the region $\{7, 11\}$ is not fixed, they form a DLS (see text).

The position of the second domain wall is not unique, however: it can be at any of the three degenerate local minima located at (6, 7), (9, 10) or (11, 12). Energetically there is no difference between these configurations, since the corresponding random-field energies are the same: $H(6) = H(9) = H(11)$. As a consequence in the interval $\{7, 11\}$ the position of the spins in the ground state is not fixed. Such a region will be referred to as a domain of 'loose' spins (DLS).

Based on this example, we can easily postulate the properties of a DLS. First let us restrict ourselves to a DLS which separates a $\downarrow \ldots \downarrow$ and a $\uparrow \ldots \uparrow$ domain, as shown in our example in figure 1. Such a DLS is bounded by two degenerate local minima. Inside the DLS the integrated random field function relative to its value at the boundaries, $\Delta H(k)$, does not exceed $2J$; thus the energy necessary to create a domain wall cannot be accumulated from the random field. On the other hand, in both directions outside the DLS, $\Delta H(k)$ exceeds $2J$ before reaching zero. This last condition ensures the existence of $\downarrow \ldots \downarrow$ and $\uparrow \ldots \uparrow$ ferromagnetic domains at two sides of the DLS.

The other type of DLS separating $\uparrow \ldots \uparrow$ and $\downarrow \ldots \downarrow$ domains can be characterized similarly. In this case, after the transformation $\sigma_i \to -\sigma_i$, $h_i \to -h_i$, the previous considerations can be extended to $-H(k)$. Note that if $2J/H$ is an integer, the degeneracy of the ground state is higher than for a slightly larger or slightly smaller value of the coupling. It is connected to the fact that the conditions both for a $\uparrow\downarrow$ and for a $\downarrow\uparrow$ DLS can be satisfied in this case at the same time. In what follows we are not going to deal with such situations, so that our analysis applies for $2J/H \neq$ integer.

Since the position of spins in a DLS is not fixed, these regions are the source of non-zero ground-state entropy in a RFIM. These regions are also responsible for a non-vanishing value of the connected correlation functions at $T = 0$. It is easy to see that the thermal averaging of $\chi(l)$ in equation (7), which is now performed over the degenerate ground-state configurations, is non-zero only if the two spins are in the same DLS. If one spin is in a ferromagnetic domain, its value is the same for all ground-state configurations; consequently, the connected correlations are zero. On the other hand, if two spins are in different DLS they are independent variables, so that again the connected correlations vanish.

The above relationship between $\chi(l)$ and DLS makes it possible to calculate the leading behaviour of the connected correlation function in a RFIM in a simple way. To do this one should (i) first consider DLS regions of length $\tilde{l} > l$ and determine the probability $W(\tilde{l})$ that a point of the line belongs to one of these DLS, (ii) find the probability that the other reference point of $\chi(l)$ is also on the same DLS and finally (iii) calculate the thermal and quenched averages in equation (7) with the condition that both endpoints of $\chi(l)$ are on the same DLS. In what follows we show that the leading behaviour of $\chi(l)$ is determined by the probability $W(l)$, which has an exponential dependence on l, whereas the probabilities indicated in (ii) and (iii) are comparatively negligible, because of the power-law dependence on l.

To calculate the probability $W(\tilde{l})$ one can use a geometrical interpretation of a DLS as a one-dimensional random walker with steps h_i/H on an interval consisting of

$$L = \left[\frac{2J}{H}\right] + 1 \tag{8}$$

points. Here $[x]$ denotes the integer part of a non-integer x. The walker starts at one endpoint of the interval and after $\tilde{l}$ steps made on the strip returns to the same endpoint. For large $\tilde{l}$ the leading behaviour of $W(\tilde{l})$ is exponential, and $W(\tilde{l})$ corresponds to the survival probability of the adsorbing random walker:

$$W(\tilde{l}) \sim \exp\left[-\tilde{l}/\xi(L)\right]. \tag{9}$$

We show next that the probabilities mentioned in (ii) and (iii) have weaker l dependence on l than the survivial probability. It is clear from a simple geometrical consideration that the probability in (ii) is at most $\sim 1/l$. On the other hand, to estimate the conditional

probability in (iii) one should first note that different ground states are characterized by one parameter, namely the position of the domain wall. Thus an average over this parameter is equivalent to the thermal end quenched averages in equation (7). Using the fact that the number of possible positions of a domain wall in a DLS of length l is proportional to l, one can estimate the conditional probability in (iii) as $\sim l^{-2}$.

The connected correlation function $\chi(l)$ is then obtained by summing the probabilities for $\tilde{l} \geqslant l$ with the following result in leading order:

$$\chi(l) \sim W(l) \sim \exp\left[-l/\xi(L)\right] \tag{10}$$

where L is given in equation (8). In the next section the correlation length $\xi(L)$ will be calculated for different random-field distributions.

3. Random field Ising models

The survival probability in equation (10) can be most easily calculated in the transfer matrix formalism [23]. In this formalism the elements of the transfer matrix $T(n, m)$, $n, m = 1, 2, ..., L$, are given as the probability of a step from a position m to n. In the RFIM language, $T(n, m) = P(h(n, m))$, where $h(n, m) = (n - m) \times H$. A matrix element of T is zero whenever the corresponding $h(n, m)$ is not contained in the set of random fields of the model. The leading eigenvalue of the transfer matrix, $\lambda(L)$, is connected to the survival probability as

$$W(l) \sim \lambda(L)^l. \tag{11}$$

The correlation length in equation (10) is thus given by

$$\xi(L) = -\frac{1}{\log \lambda(L)}. \tag{12}$$

For a general RFIM, simple analytical results can be obtained for $2J/H < 1$ and in the strong-coupling limit $2J/H \gg 1$. In the former case $L = 1$, the interval of the walker consists of a single point:

$$\lambda(1) = P(0) \qquad 2J/H < 1. \tag{13}$$

Therefore, non-vanishing correlations can only be present in diluted models (see e.g. equations (5) and (6)). In the strong-coupling limit, which corresponds to $L \gg 1$, we consider distributions with zero average $\langle h_i \rangle = 0$. Then the finite-size corrections to the leading eigenvalue are quadratic,

$$1 - \lambda(L) \sim L^{-2} \sim (H/J)^2 \qquad H/J \ll 1 \tag{14}$$

which follows from the Gaussian nature of the free random walk [23]. According to equations (12) and (14),

$$\xi(J/H) \sim (J/H)^2 \qquad J/H \gg 1. \tag{15}$$

For intermediate (non-integer) values of $2J/H$, the leading eigenvalue of the transfer matrix can be calculated numerically, so that in principle one can obtain the correlation length of connected correlations for all types of RFIMs, together with the asymptotic relation in equations (15). In the following we present three examples, in which the calculation can be performed analytically for all non-integer values of $2J/H$.

3.1. Binary distribution

The transfer matrix corresponding to the distribution in equation (3) is tridiagonal and given by

$$T_1 = \begin{pmatrix} 0 & p & & & & \\ q & 0 & p & & & \\ & q & 0 & p & & \\ & & & \ddots & & \\ & & & & 0 & p \\ & & & & q & 0 \end{pmatrix}. \tag{16}$$

The (non-normalized) leading right eigenvector of this matrix is

$$\Phi_1(l) = \left(\frac{q}{p}\right)^{l/2} \sin\left(l\frac{\pi}{L+1}\right) \tag{17}$$

and the corresponding leading eigenvalue is

$$\lambda_1(L) = 2\sqrt{pq}\cos\left(\frac{\pi}{L+1}\right). \tag{18}$$

The correlation length is then can be obtained from equations (12) and (8). This result agrees with that of [16].

3.2. Diluted symmetric binary distribution

The transfer matrix corresponding to the distribution in equation (5) is symmetric and tridiagonal:

$$T_2 = \begin{pmatrix} q & p/2 & & & & \\ p/2 & q & p/2 & & & \\ & p/2 & q & p/2 & & \\ & & & \ddots & & \\ & & & & p/2 & \\ & & & & p/2 & q \end{pmatrix}. \tag{19}$$

The leading eigenvector of T_2 is the same as in equation (17), but $q/p = 1$ and the leading eigenvalue is given by

$$\lambda_2(L) = q + p\cos\left(\frac{\pi}{L+1}\right). \tag{20}$$

For $L = 1$, i.e. for $2J/H < 1$, $\lambda_2(1) = q$ and the general relation in equation (13) is recovered, which also agrees with the result of Grinstein and Mukamel [9] for $2J/H \to 0$.

3.3. Diluted symmetric exponential distribution

We consider the discretized version of the distribution in equation (6) when the allowed values of the random field are $h_i = H \times i$, $i = 0, \pm 1, \pm 2, \ldots$ and the probability distribution is given by

$$P(h) = c \exp\left(-\alpha \left|\frac{h}{H}\right|\right) \delta(h - Hi) + c_0 \delta(h). \tag{21}$$

Here $\alpha > 0$ and the distribution is normalized with $c = p/2(\exp\alpha - 1)$, $c_0 = q - c$ and $p + q = 1$. Taking the limit

$$\alpha \to 0^+ \qquad H \to 0 \qquad \tilde{H} = H/(\exp\alpha - 1) = \text{finite} \tag{22}$$

one arrives at a continuum description in the variables $h_i = \tilde{H} x_i$, $-\infty < x_i < \infty$, with the probability distribution $R(x)dx$ given by equation (6).

We write the transfer matrix of the problem in terms of the variables $\omega = \exp(-\alpha)$, $\omega_0 = q$ and $\kappa = c/q$ as

$$T_3 = \omega_0 \begin{pmatrix} 1 & \kappa\omega & \kappa\omega^2 & \ldots & \kappa\omega^{L-1} \\ \kappa\omega & 1 & \kappa\omega & & \\ \kappa\omega^2 & \kappa\omega & 1 & & \\ \vdots & & & \ddots & \\ \kappa\omega^{L-1} & \ldots & & & 1 \end{pmatrix}. \tag{23}$$

We mention that the same transfer matrix belongs to a directed polymer [23] on a strip of width L on the square lattice. In this case ω_0 and ω are the monomer fugacities for steps along and perpendicularly to the strip, respectively, and κ denotes the statistical weight corresponding to a bend of the chain by 90°.

The eigenvalue problem of T_3 in equation (23) is similar to that of the unrestricted directed polymer [24]. The leading eigenvector is given by

$$\Phi_3(l) = \cos\left[\left(l - \frac{L+1}{2}\right)\phi\right] \tag{24}$$

where ϕ is the smallest root of the equation

$$\cot\left(\frac{(L-1)\phi}{2}\right) = \frac{\sin\phi}{\cos\phi - \omega}. \tag{25}$$

The leading eigenvalue is then

$$\lambda_3(L) = \frac{(1-\omega^2)\kappa\omega_0}{1 - 2\omega\cos\phi + \omega^2} + (1-\kappa)\omega_0. \tag{26}$$

The correlation length can be obtained from equation (12) using the correspondences between ω, ω_0, κ and the original parameters of the distribution in equation (21).

Now we evaluate our results in equations (25) and (26) in the continuum limit of equation (22), which reads as $\omega \to 1^-$. The smallest root of equation (25) is proportional

to $1-\omega$, so that writing it in the form $\phi = \gamma(1-\omega)$ one derives the following relation from equation (25):

$$\tan^{-1}\left(\frac{1}{\gamma}\right) = \frac{J}{H}\gamma. \tag{27}$$

Then the leading eigenvalue in the continuum limit is given by

$$\lambda_3 = \frac{1+q\gamma^2}{1+\gamma^2}. \tag{28}$$

These results are identical to those obtained by Luck and Nieuwenhuizen [12] using the continuous distribution in equation (6) and taking the non-trivial limit $T \to 0^+$. We note that the correlation function calculated with the continuous distribution is discontinuous at $T = 0$, so that the limits $H \to 0$ and $T \to 0$ can not be interchanged.

4. Random-bond Ising model

The formalism developed in section 2 can also be applied to calculate $\chi(l)$ for the RBIM with the Hamiltonian in equation (1). It is easy to see that after the transformation $\sigma_i \to \sigma_i h/h_i$ in equation (2) one arrives at a RBIM with $J_{i,i+1} = J/h_i h_{i+1}$. The inverse transformation is given by

$$\sigma_i \to \sigma_i \prod_{k=1}^{i-1}\left(\frac{J_{k,k+1}}{J}\right) \tag{29}$$

so that random fields can be expressed via random bonds as

$$h_i = h\prod_{k=1}^{i-1}\left(\frac{J_{k,k+1}}{J}\right). \tag{30}$$

The random fields in equation (30) can assume the values $\pm h$, like the binary distribution in equation (3); however, these h_is are correlated in different sites, since

$$\langle h_i h_{i+n}\rangle = (p-q)^n. \tag{31}$$

Thus the symmetric distribution with $p = q = 1/2$ is exceptional, in which case the RBIM is equivalent to the RFIM with the symmetric binary distribution of equation (3).

For $p \neq q$ the probability is connected to bond variables, thus to the sign of the product of two consecutive random fields:

$$P(h_{i-1}h_i) = \begin{cases} p & h_{i-1}h_i > 0 \\ q & h_{i-1}h_i < 0 \end{cases}. \tag{32}$$

To write down the transfer matrix of this problem we work with the bond variable $\bar{H}(i) = (H(i-1)+H(i))/2$, which may take $L-1$ different values, $1/2, 3/2, \ldots, L-1/2$.

The one-step transfer matrix $T_{RB}(i, i+1)$ is different for even and odd numbers i of the step. Therefore one should consider the two-step transfer matrix defined as

$$T_{\mathrm{RB}}(i, i+2) = \begin{pmatrix} q^2 & qp & p^2 & & & & & \\ qp & q^2 & qp & & & & & \\ & qp & q^2 & qp & p^2 & & & \\ & p^2 & qp & q^2 & qp & & & \\ & & & qp & q^2 & qp & p^2 & \\ & & & p^2 & qp & q^2 & qp & \\ & & & & & & \ddots & \\ & & & & & & & \ddots \end{pmatrix}. \tag{33}$$

The transfer matrix connecting the states in steps $i+1$ and $i+3$ is the transpose of $T_{\mathrm{RB}}(i, i+2)$, so that both have the same eigenvalue spectrum. Writing the leading eigenvalue as λ_{RB}^2 ,the correlation length is obtained from the logarithm of λ_{RB} via equation (12). Since the transfer matrix in equation (33) is non-symmetric and non-tridiagonal, its eigenvalue problem is solvable analytically only in a few special cases.

In the limit $p \to 0$ the transfer matrix is tridiagonal in linear order of p and can be solved by the same eigenvector as T_2 of equation (19). The leading eigenvalue is given by

$$\lambda_{\mathrm{RB}} = 1 - \frac{p}{2}\left(1 - \cos\frac{\pi}{L}\right) + \mathrm{O}(p^2). \tag{34}$$

In the symmetric distribution $p = q = 1/2$ the right eigenvector is given by

$$\Phi_{\mathrm{RB}}(2l) = \Phi_{\mathrm{RB}}(2l+1) = \sin\left[(2l+1)\frac{\pi}{L+1}\right] \tag{35}$$

for $l = 0, 1, ..., L/2 - 1$. The corresponding eigenvalue

$$\lambda_{\mathrm{RB}} = \cos\left(\frac{\pi}{L+1}\right) \qquad (p = q = 1/2) \tag{36}$$

is the same as for the RFIM with symmetric distribution in equation (18), which is in accord with our previous claim about the equivalence of the two problems.

Finally, we consider the limit $q \to 0$. The eigenvector is then given in leading order by

$$\begin{aligned} \Psi(2l) &= \Psi(L - 2l + 1) = q^{(l-1)/(L-1)} \\ \Psi(2l-1) &= \Psi(L - 2l + 2) = q^{1-l/(L-1)} \end{aligned} \tag{37}$$

for $l = 1, 2, ..., (L-1)/2$. The corresponding eigenvalue is

$$\lambda_{\mathrm{RB}} = q^{1/L} \qquad q \ll 1. \tag{38}$$

Analysing the p-dependence of the correlation length, one can see that it starts with zero in the ferromagnetic limit $p \to 1$, stays finite for non-zero concentration of ferromagnetic bonds and finally diverges in the antiferromagnetic limit $p \to 0$.

5. Discussion

In this paper the connected correlation function of random Ising chains is investigated at zero temperature. Our study is based on an analysis of the ground-state configurations of these systems. Due to frustration, there are spins in the ground state which are 'loose', i.e. their position is not fixed by the interaction and the external field. These 'loose' spins form domains, and between two spins in the same domain there are non-vanishing correlations. It was shown then that the connected correlation function $\chi(l)$ is primarily determined by the probability of having a DLS of size l in the system. Finally, this probability was calculated using an analogy with the survival probability of some random walker on a finite interval.

Considering different types of random-field and random-bond distributions, we have calculated the correlation length using the transfer matrix method. Analysing these results one my observe two different types of behaviour in the weak disorder limit. The correlation length either vanishes as $\xi \sim 1/\log(1/p)$ or is divergent as $\xi \sim 1/p$. The former behaviour is found in the RFIM with binary distribution in equation (18), as well as for the RBIM in the pure ferromagnetic limit $q \to 0$ in equation (38). On the other hand, the correlation length is diverging in the diluted RFIM in equations (20) and (28) as $p \to 0$. Similar p-dependence is observed in the RBIM in the antiferromagnetic limit, i.e. when $p \to 0$ in equation (34).

The exponential decay of correlations for random Ising chains is found as a general rule. One may find, however, a slower decay of correlations if the strength of randomness is smoothly position-dependent. Let us consider a semi-infinite RFIM in which the strength of the random field decays as $h_i \sim i^{-s}$ from the surface. Then the equivalent random walker has to be considered on an interval the size of which is increasing in time. Using results about random walkers [25] and directed polymers [24] inside a parabola, one can say that the decay of correlations in this inhomogeneuosly disordered system is of a stretched exponential form for $0 < s < 2$, whereas it can be described as a power law for $s \geqslant 2$.

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References

[1] Nattermann T and Villain J 1988 *Phase Transitions* **11** 5
[2] Nattermann T and Rujan P 1989 *Int. J. Mod. Phys.* B **3** 1597
[3] Luck J M 1992 *Systémes Désordonnés Unidimensionels* (Collection Aléa-Saclay)
[4] Toulouse G 1977 *Commun. Phys.* **2** 115
[5] Fishman S and Aharony A 1979 *J. Phys. C: Solid State Phys.* **12** L729
[6] Derrida B, Vannimenus J and Pomeau Y 1978 *J. Phys. C: Solid State Phys.* **11** 4749
[7] Williams J K 1981 *J. Phys. C: Solid State Phys.* **14** 4095
[8] Behn U, Priezzhev V B and Zagrebnov A 1990 *Physica* **167A** 481
[9] Grinstein G and Mukamel D 1983 *Phys. Rev.* B **27** 4503
[10] Amic E and Luck J M 1994 *Preprint* SPhT/94/005 Saclay
[11] Nieuwenhuizen Th M and Luck J M 1986 *J. Phys. A: Math. Gen.* **19** 1207
[12] Luck J M and Nieuwenhuizen Th M 1989 *J. Phys. A: Math. Gen.* **22** 2151

[13] Luck J M, Funke M and Nieuwenhuizen Th M 1991 *J. Phys. A: Math. Gen.* **24** 4155
[14] Derrida B 1983 *Les Houches Winter School Proceedings*
[15] Derrida B and Hilhorst H J 1981 *J. Phys. C: Solid State Phys.* **14** L539
[16] Farhi E and Gutmann S 1993 *Phys. Rev.* B **48** 9508
[17] Fonk Y and Hilhorst H J 1987 *J. Stat. Phys.* **49** 1235
[18] Stephenson J 1970 *J. Math. Phys.* A **11** 413
[19] Forgacs G 1980 *Phys. Rev.* B **22** 4473
[20] Peschel I 1982 *Z. Phys.* B **45** 339
[21] Wolff W F and Zittartz J 1982 *Z. Phys.* B **47** 341
[22] Sütő A 1981 *Z. Phys.* B **44** 121
[23] Privman V and Švrakić N M 1989 Directed models of polymers, interfaces and finite-size properties (Lecture Notes in Physics 338) (Berlin: Springer)
[24] Iglói F 1992 *Phys. Rev.* A **45** 7024
[25] Turban L 1992 *J. Phys. A: Math. Gen.* **25** L127